

Medium Modification of Pion Structure

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The Collaboration



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Hyak @ UW



Sporades @ W&M

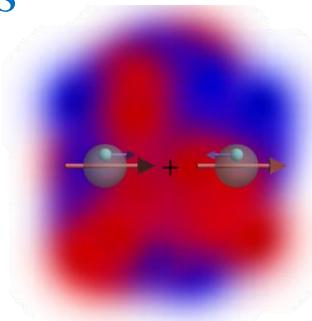


Hopper @ NERSC

Medium Modification

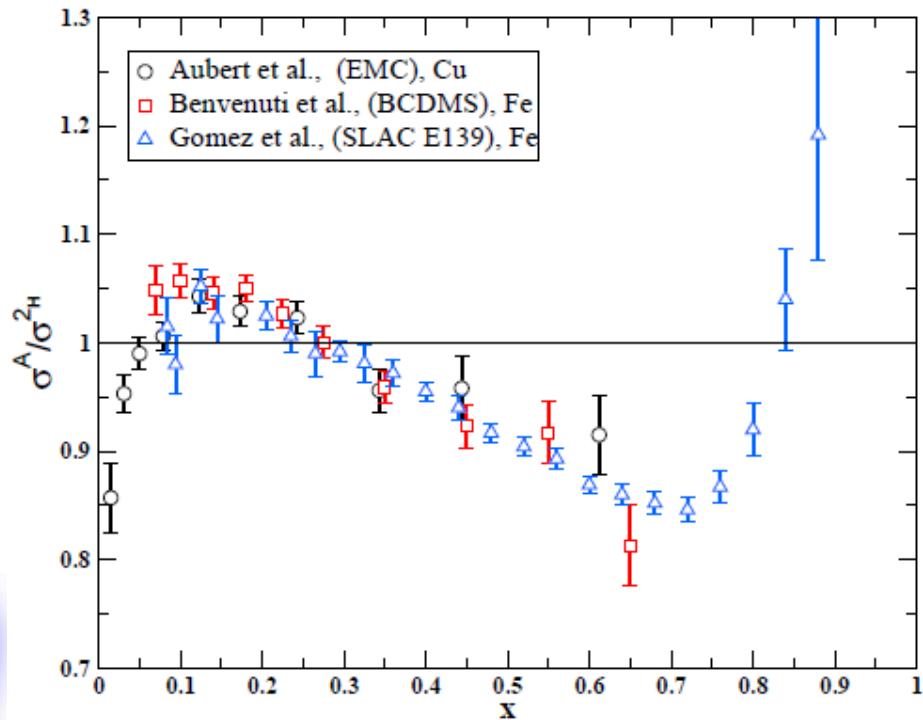
§ Parton structure function in nuclear medium

- ❖ The famous EMC effect
- ❖ Significant deviations between heavy nuclei and deuterium
- ❖ Many models:
 - pion enhancement
 - nucleon expansion
 - multiquark clusters
 - rescaling
 - shadowing
 - local correlations



...

§ No universal understanding

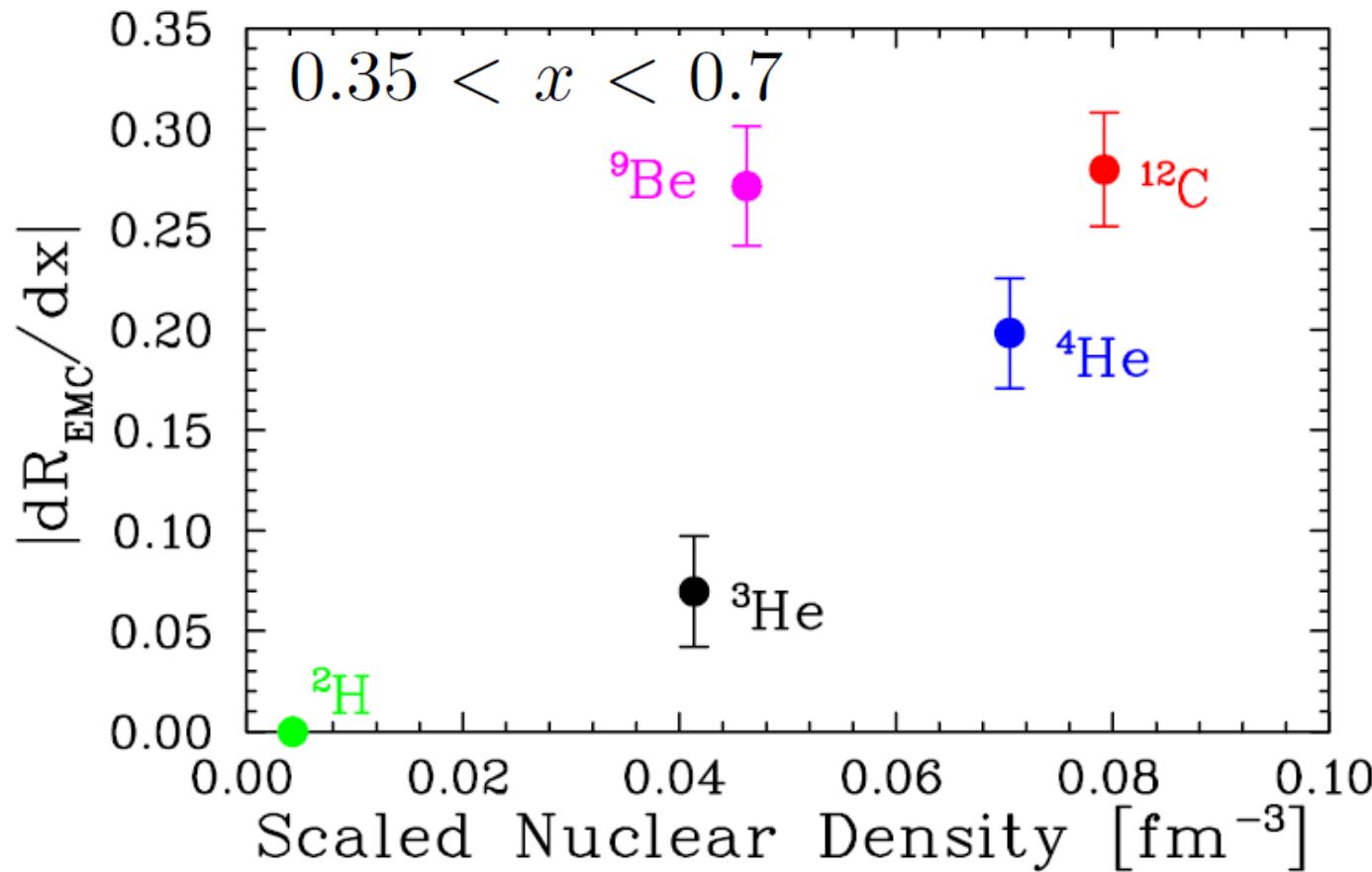


J. J. Aubert et al.
Phys. Lett. 123, 275 (1983)

Medium Modification

§ Not only significant for heavy nuclei,
also important for light-nuclear systems

J. Seely et al., Phys. Rev. Lett. 103, 202301 (2009)

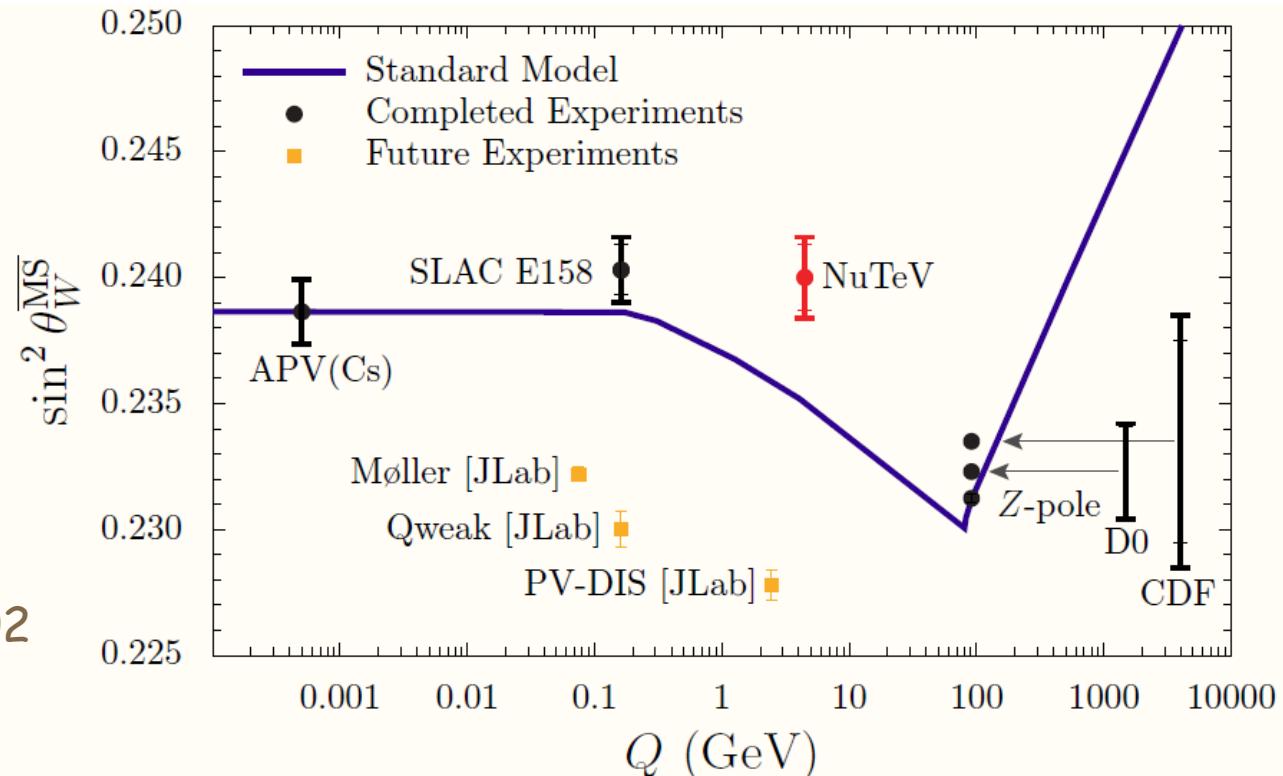


Applications

§ Important for tests of Standard Model

§ For example, NuTeV anomaly

- ❖ Weak mixing angle experiment
- ❖ 3 sigma away from SM → “NuTeV anomaly”



G. P. Zeller et al.
Phys. Rev. Lett. 88, 091802
(2002)

Applications

§ Important for tests of Standard Model

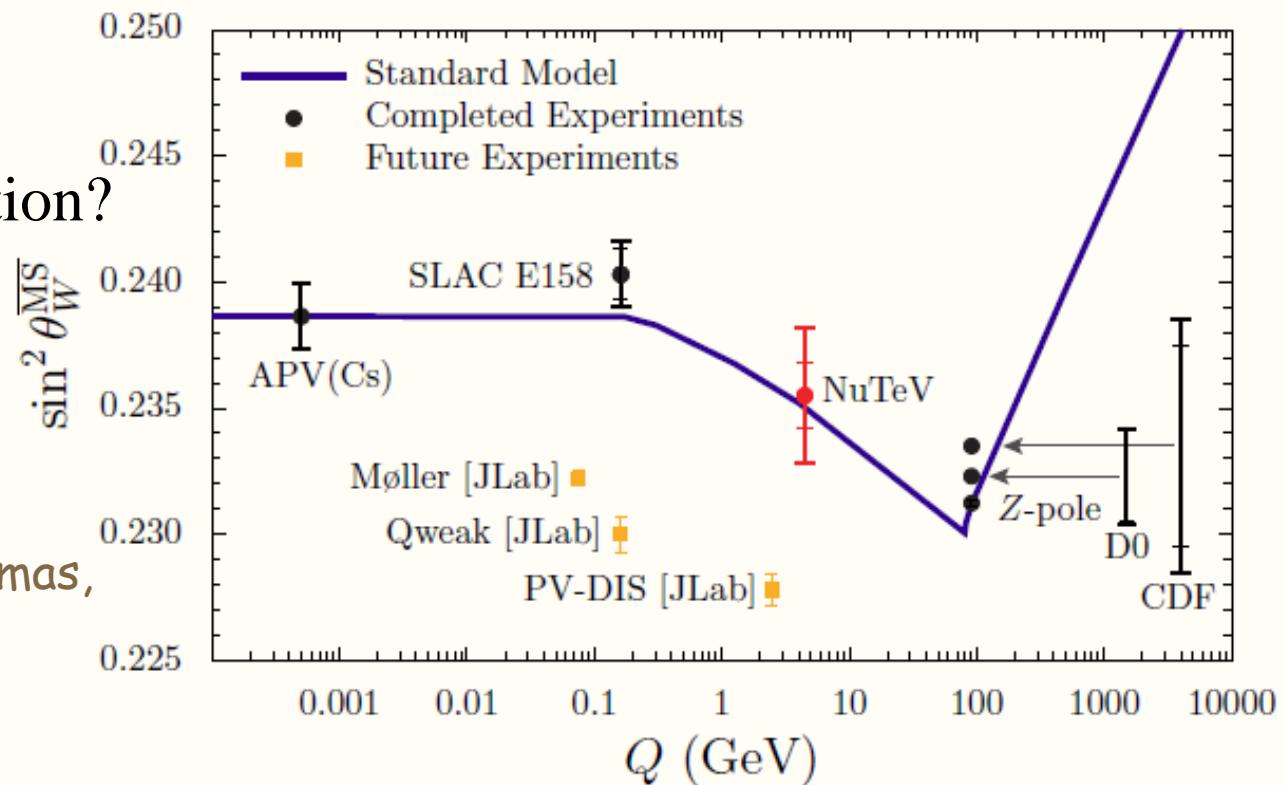
§ For example, NuTeV anomaly

- ❖ Weak mixing angle experiment
- ❖ 3 sigma away from SM → “NuTeV anomaly”

❖ Evidence for medium modification?

❖ Correction looks consistent with SM

I. Cloët, W. Bentz, A. Thomas,
Phys. Lett. B693, 462
(2010)



Medium Modification

§ Interesting physics with multi-baryon systems

- ❖ Study with nonzero μ_B
notorious “sign” problem in Monte Carlo calculation
- ❖ Add nucleons to the system
complicated quark contraction and noise/signal issue

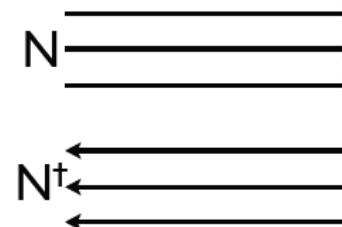
Number of contractions:

$$(A+Z)! \times (2A-Z)!$$

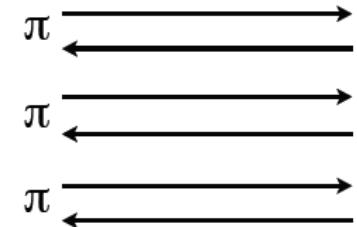
- ❖ Triton: $2880 \rightarrow 93$
- ❖ ${}^4\text{He}$: $518400 \rightarrow 1107$

Signal Quality

- ❖ What you want:



- ❖ What you get:



§ Take a step back and look at nonzero-isospin/multi-meson systems

- ❖ Many studies: “QCD at Finite Isospin Density”, Son & Stephanov
- ❖ Gain insight and experience for how to deal with nuclear systems

We do it all the time...



We do it all the time...



We do it all the time...



Multi-Meson System

§ First lattice π - π calculation in '92

§ Meson number > 2 system is first studied on the lattice

S. Beane et al., PRL 100: 082004 (2008)

§ Followed by more complicated systems for scattering length and 3-body interactions

W. Detmold et al., PRD 78: 014507 (2008); W. Detmold et al., PRD 78: 054514 (2008); W. Detmold et al., PRL 102: 032004 (2009); W. Detmold et al., 1103.4362 (2011)

§ Effective theory

S. Beane, PRD 76: 074507 (2007); W. Detmold et al., PRD 77: 057502 (2008); Smigielski, PRD 79: 054506 (2009)

§ Color-screening effects

W. Detmold et al., PRL 102: 032004 (2009)

§ Study for $n \geq 13$ system

W. Detmold et al., PRD 82: 014511 (2010); Z. Shi and W. Detmold, PoS (2011)

Boring Details

§ The tool: Lattice gauge theory

§ The actions

❖ Domain-wall fermions on staggered sea

- ❖ Free forward propagators from LHPC and NPLQCD
- ❖ Chiral symmetry preserved at finite lattice spacing;
good for spin physics and weak matrix elements

§ The parameters

Label	a [fm]	$L^3 \times T$	m_π [MeV]	$m_\pi L$	$m_\pi T$	Measurements
F	0.09	$28^3 \times 96$	320	4.1	14.0	432
C1	0.12	$20^3 \times 64$	290	3.7	11.7	1450
C2	0.12	$20^3 \times 64$	350	4.4	14.2	837
C3	0.12	$20^3 \times 64$	490	6.2	19.9	1000

t_{sep} C's $\in \{16, 20, 24, 28, 32\}$
 F's $\in \{24, 32, 48\}$

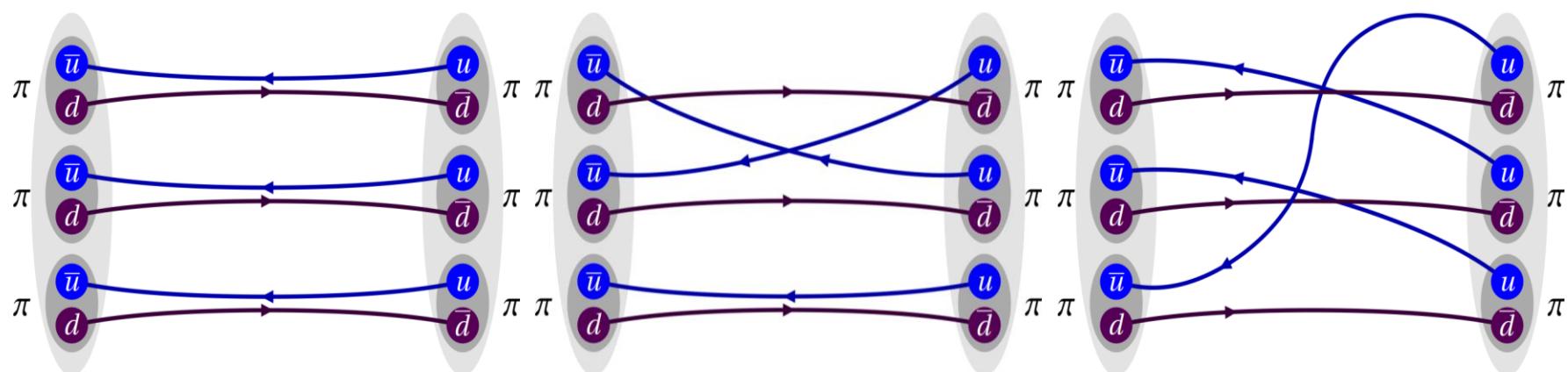
$\pi^+ \text{ in Medium}$

§ We calculate

$$C_m(t, \mathbf{p}) = \left\langle 0 \left| \left(\prod_{i=1}^m \sum_{\mathbf{x}} e^{i \mathbf{p}_i \cdot \mathbf{x}} \pi^+(\mathbf{x}, t) \right) (\pi^-(x_0))^m \right| 0 \right\rangle \quad \mathbf{p} = \sum_{i=1}^m \mathbf{p}_i$$

§ Contractions

❖ Examples from 3- π^+ system



$\pi^+ \text{ in Medium}$

§ We calculate

$$C_m(t, \mathbf{p}) = \left\langle 0 \left| \left(\prod_{i=1}^m \sum_{\mathbf{x}} e^{i \mathbf{p}_i \cdot \mathbf{x}} \pi^+(\mathbf{x}, t) \right) (\pi^-(x_0))^m \right| 0 \right\rangle \quad \mathbf{p} = \sum_{i=1}^m \mathbf{p}_i$$

§ Contractions

❖ Simplified with recursion relation

W. Detmold et al., Phys.Rev.D82:014511 (2010)

$$C_3(t) = \text{tr}[\Pi]^3 - 3 \text{tr}[\Pi] \text{tr}[\Pi^2] + 2 \text{tr}[\Pi^3]$$

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x}, t; 0) \gamma_5 S^\dagger(\mathbf{x}, t; 0)$$

§ Energy extraction from n - π^+ system

$$C_m(t, \mathbf{0}) \rightarrow \sum_{\ell=0}^m \binom{m}{\ell} Z_m^{(\ell)} e^{-E_{m-\ell} t} e^{-E_\ell(T-t)} + \dots$$

n - π^+ Energy

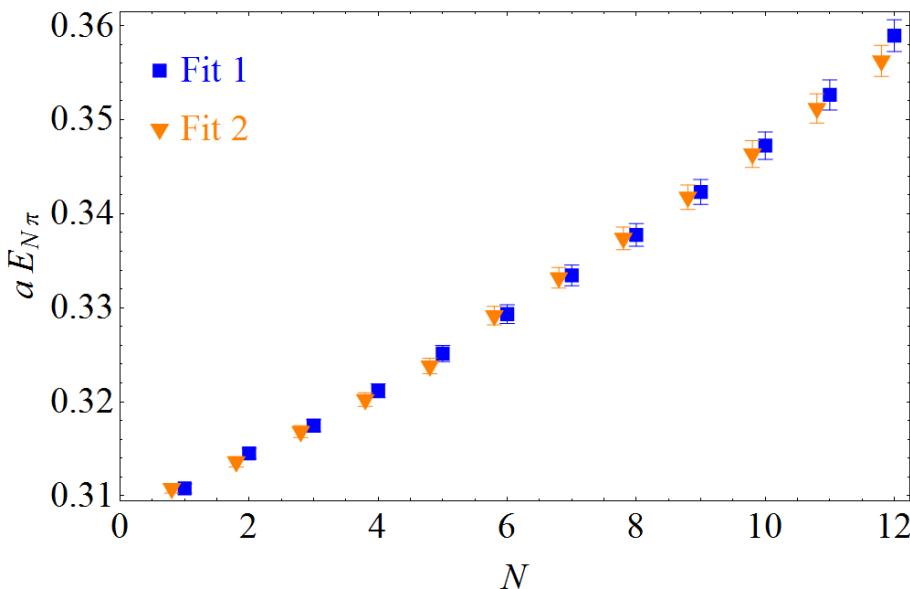
§ Can we simplify the analysis?

❖ Examples from n - π^+ system

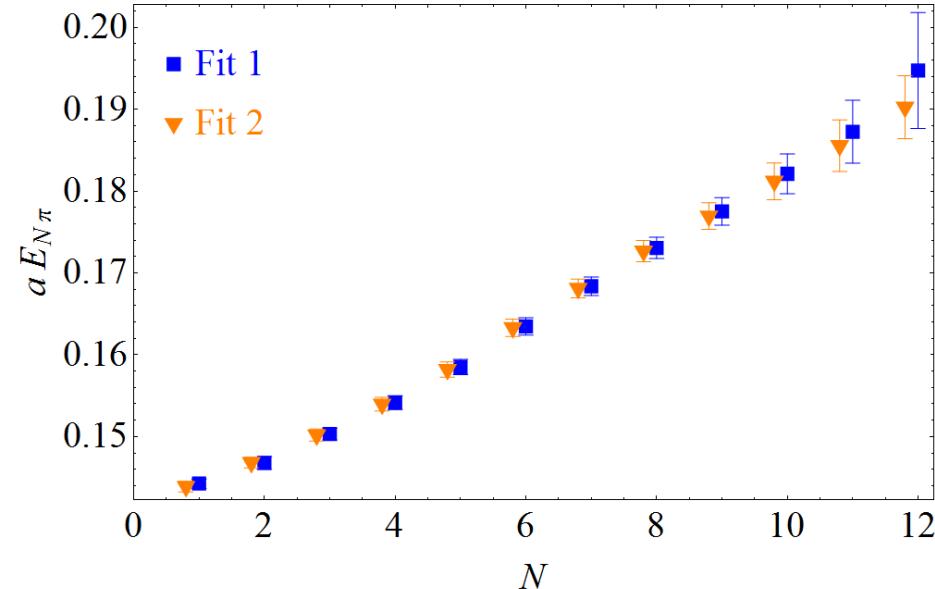
Fit 1: $C_m(t, \mathbf{0}) \rightarrow \sum_{\ell=0}^m \binom{m}{\ell} Z_m^{(\ell)} e^{-E_{m-\ell} t} e^{-E_\ell(T-t)} + \dots$

Fit 2: $\ell = m$ only (ignore the thermal states)

$a = 0.12 \text{ fm}, M_\pi \approx 490 \text{ MeV}$



$a = 0.09 \text{ fm}, M_\pi \approx 320 \text{ MeV}$

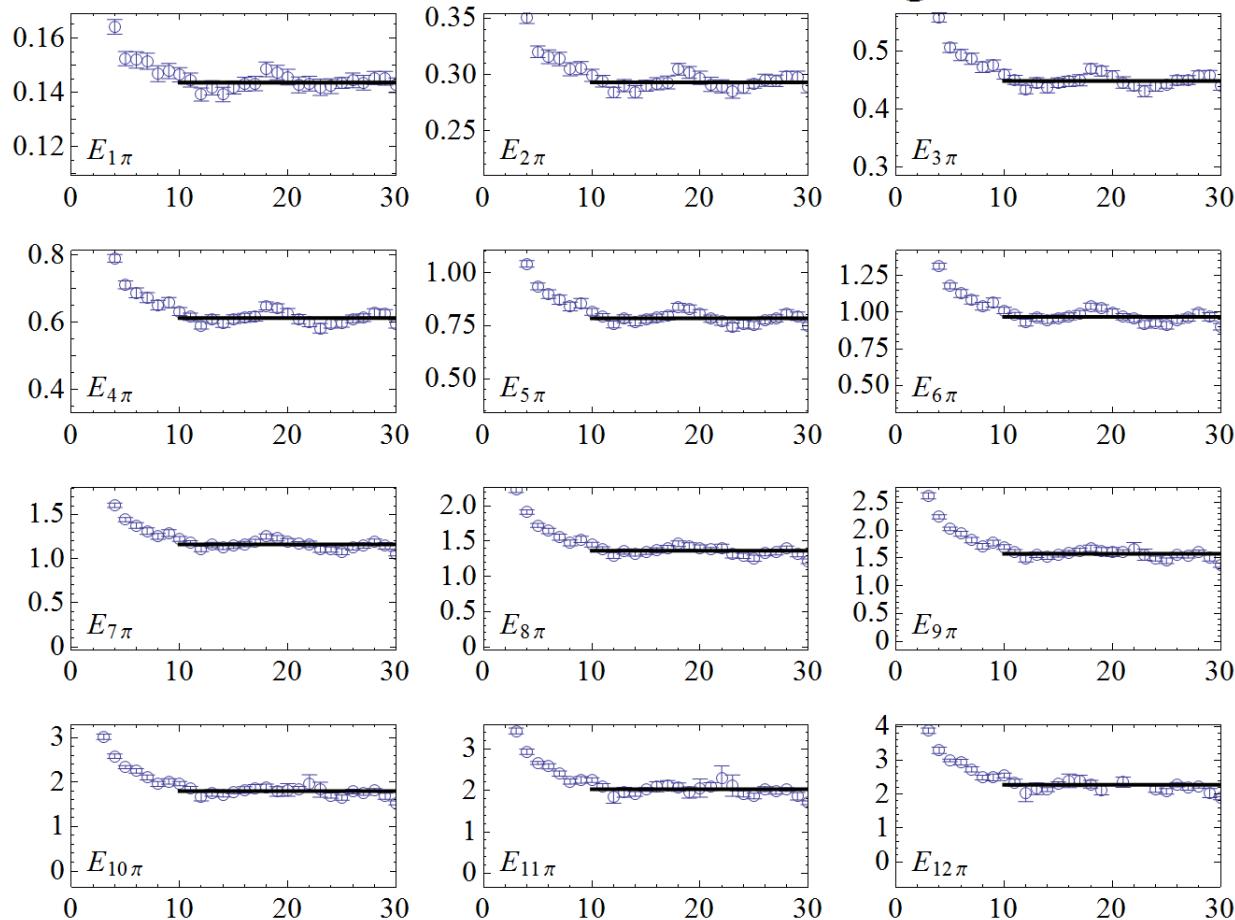


$n\text{-}\pi^+$ Energy

§ Fitted energies and effective-mass plot

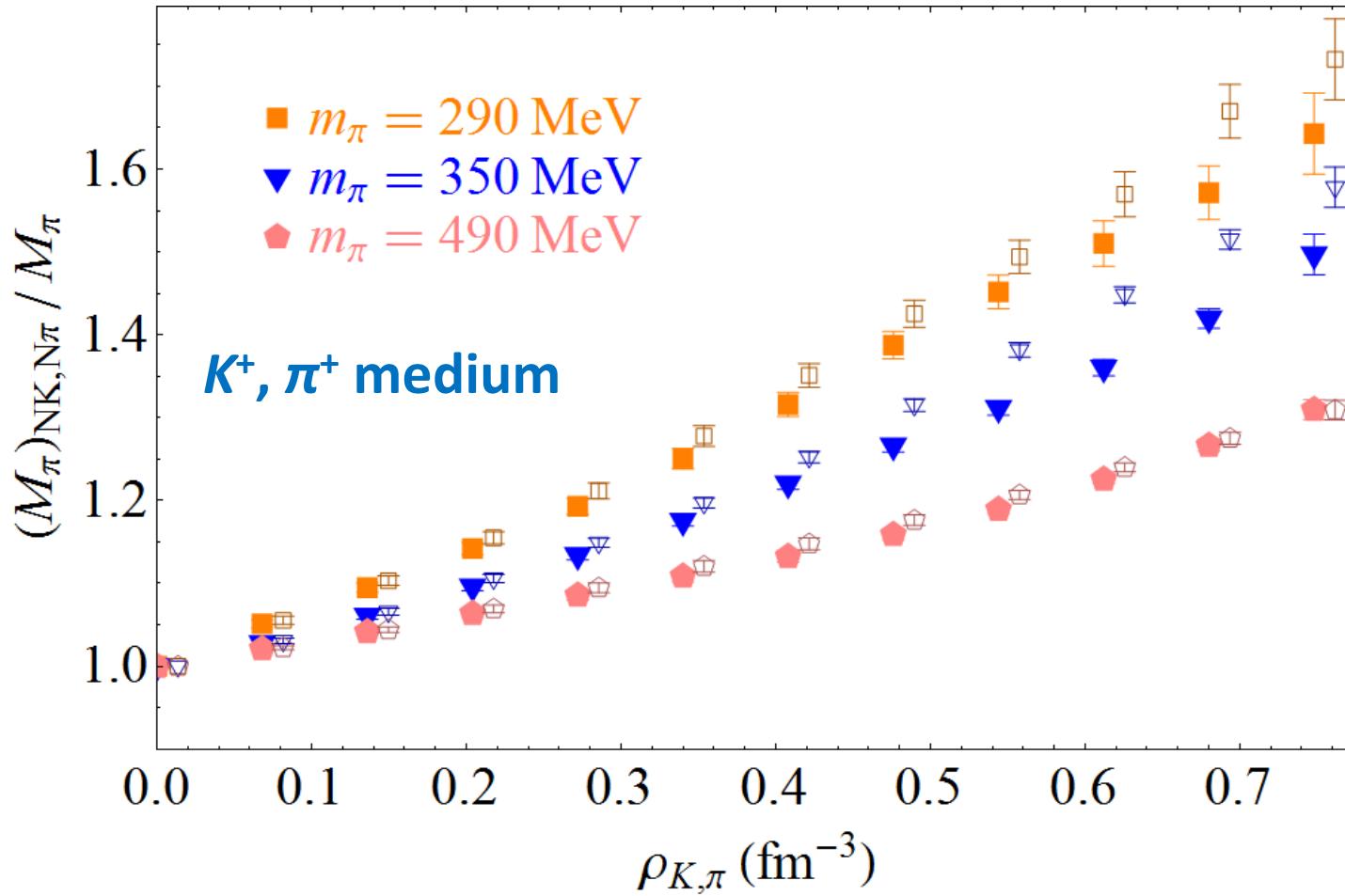
❖ Examples

$a = 0.09 \text{ fm}$,
 $M_\pi \approx 320 \text{ MeV}$



\mathcal{M}_π in Medium

§ How pion mass shifts in K^+ , π^+ medium



Pion Structure Function

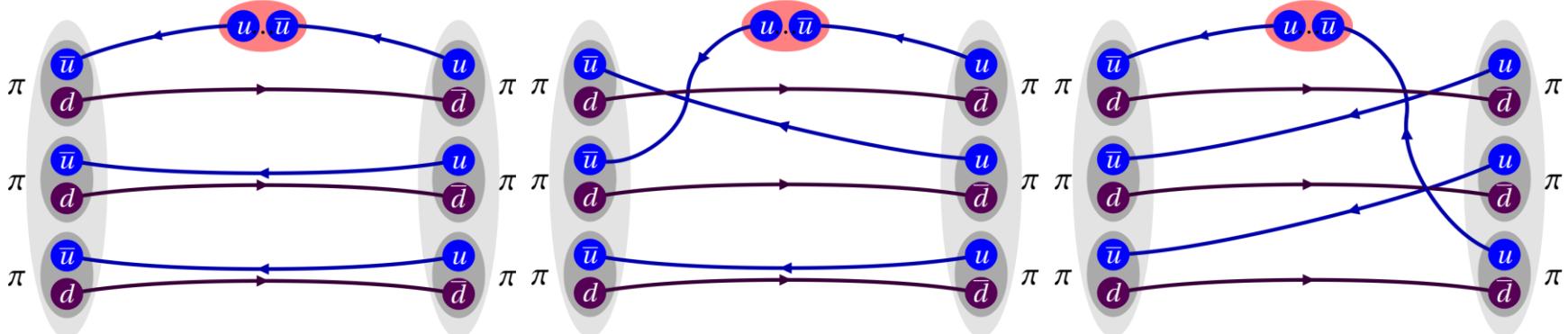
§ We calculate

$$C_m^{(n)}(\tau, t, \mathbf{p}) = \left\langle 0 \left| \left(\prod_{i=1}^m \sum_{\mathbf{x}} e^{i \mathbf{p}_i \cdot \mathbf{x}} \pi^+(\mathbf{x}, t) \right) \left(\sum_{\mathbf{y}} O_{\{\mu_0 \mu_1 \dots \mu_n\}}(\mathbf{y}, \tau) \right) (\pi^-(x_0))^m \right| 0 \right\rangle$$

where $O_{\{\mu_0 \mu_1 \dots \mu_n\}} = \left(\frac{i}{2}\right)^n \bar{\psi} \gamma_{\mu_0} \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi$ – trace

§ Contractions

❖ Examples from 3- π^+ system



Pion Structure Function

§ We calculate

$$C_m^{(n)}(\tau, t, \mathbf{p}) = \left\langle 0 \left| \left(\prod_{i=1}^m \sum_{\mathbf{x}} e^{i \mathbf{p}_i \cdot \mathbf{x}} \pi^+(\mathbf{x}, t) \right) \left(\sum_{\mathbf{y}} O_{\{\mu_0 \mu_1 \dots \mu_n\}}(\mathbf{y}, \tau) \right) (\pi^-(x_0))^m \right| 0 \right\rangle$$

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§ Wick contractions

❖ Use mixed-meson recursion relations

W. Detmold, B. Smigielski, Phys.Rev.D84:014508 (2011)

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x}, t; 0) \gamma_5 S^\dagger(\mathbf{x}, t; 0)$$

$$\tilde{\Pi} = \sum_{\mathbf{x}, \mathbf{y}} \gamma_5 S(\mathbf{x}, t; \mathbf{y}, \tau) O S(y, \tau; 0) \gamma_5 S^\dagger(\mathbf{x}, t; 0) \rightarrow \text{Treated as diff. meson splices}$$

Pion Structure Function

§ We calculate

$$C_m^{(n)}(\tau, t, \mathbf{p}) = \left\langle 0 \left| \left(\prod_{i=1}^m \sum_{\mathbf{x}} e^{i \mathbf{p}_i \cdot \mathbf{x}} \pi^+(\mathbf{x}, t) \right) \left(\sum_{\mathbf{y}} O_{\{\mu_0 \mu_1 \dots \mu_n\}}(\mathbf{y}, \tau) \right) (\pi^-(x_0))^m \right| 0 \right\rangle$$

$$\text{where } O_{\{\mu_0 \mu_1 \dots \mu_n\}} = \left(\frac{i}{2} \right)^n \bar{\psi} \gamma_{\mu_0} \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi - \text{trace}$$

§ Wick contractions

❖ Examples from n - π^+ system

$$C_3^{(n)} = -2 \text{tr}[\Pi] \text{tr}[\tilde{\Pi} \Pi] + 2 \text{tr}[\tilde{\Pi} \Pi^2] + \text{tr}[\Pi]^2 \text{tr}[\tilde{\Pi}] - \text{tr}[\Pi^2] \text{tr}[\tilde{\Pi}]$$

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x}, t; 0) \gamma_5 S^\dagger(\mathbf{x}, t; 0), \quad \tilde{\Pi} = \sum_{\mathbf{x}, \mathbf{y}} \gamma_5 S(\mathbf{x}, t; \mathbf{y}, \tau) O S(\mathbf{y}, \tau; 0) \gamma_5 S^\dagger(\mathbf{x}, t; 0)$$

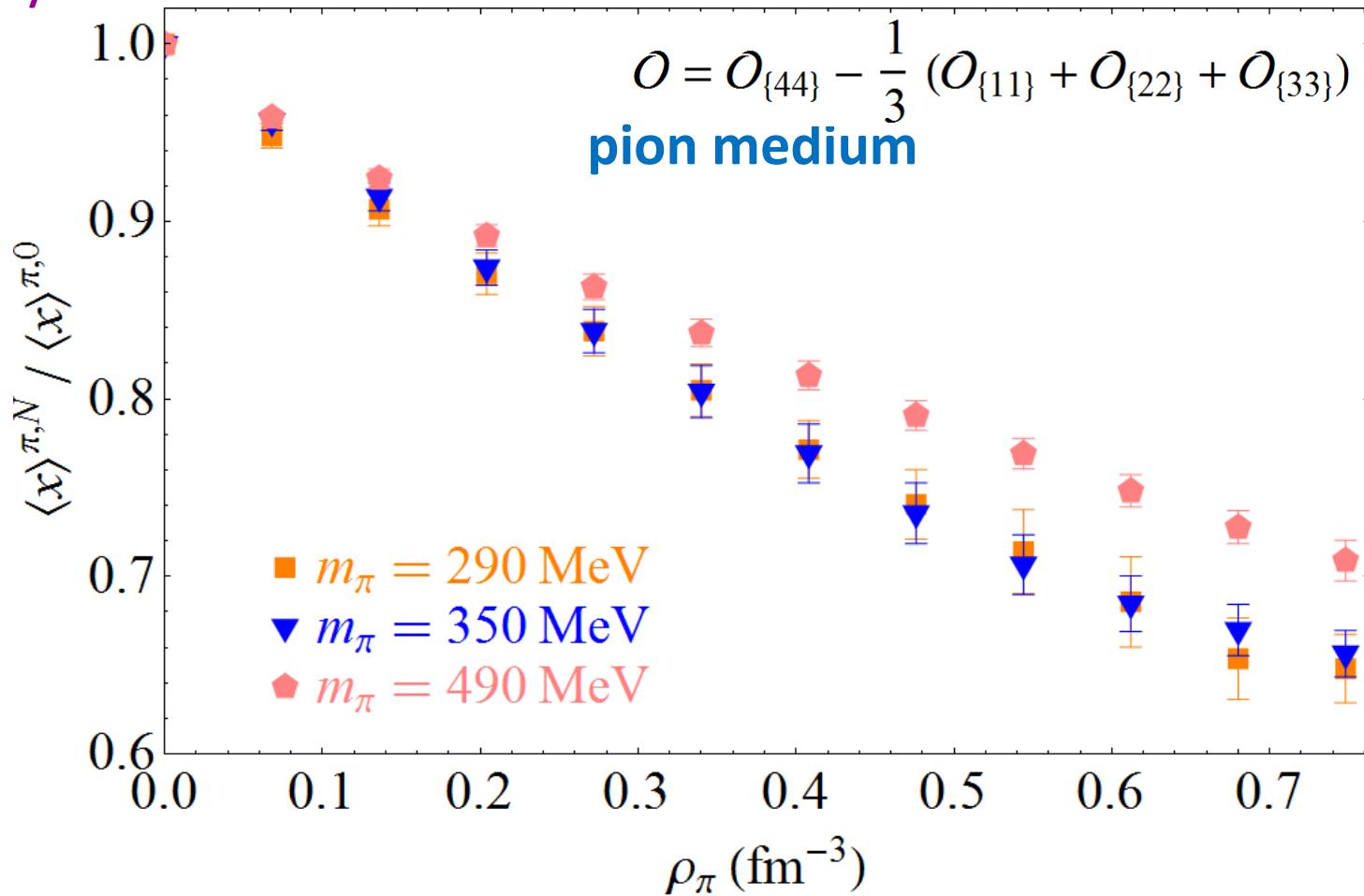
§ Matrix elements extraction (naively)

$$C_m^{(n)}(\tau, t, \mathbf{p}) \xrightarrow{t \gg \tau \gg 0} A e^{-E_n t} \langle n\pi | O | n\pi \rangle + \dots$$

Pion Momentum Fraction

§ $\langle x \rangle^{\pi, N} / \langle x \rangle^{\pi, 0}$ without thermal-state degrees of freedom

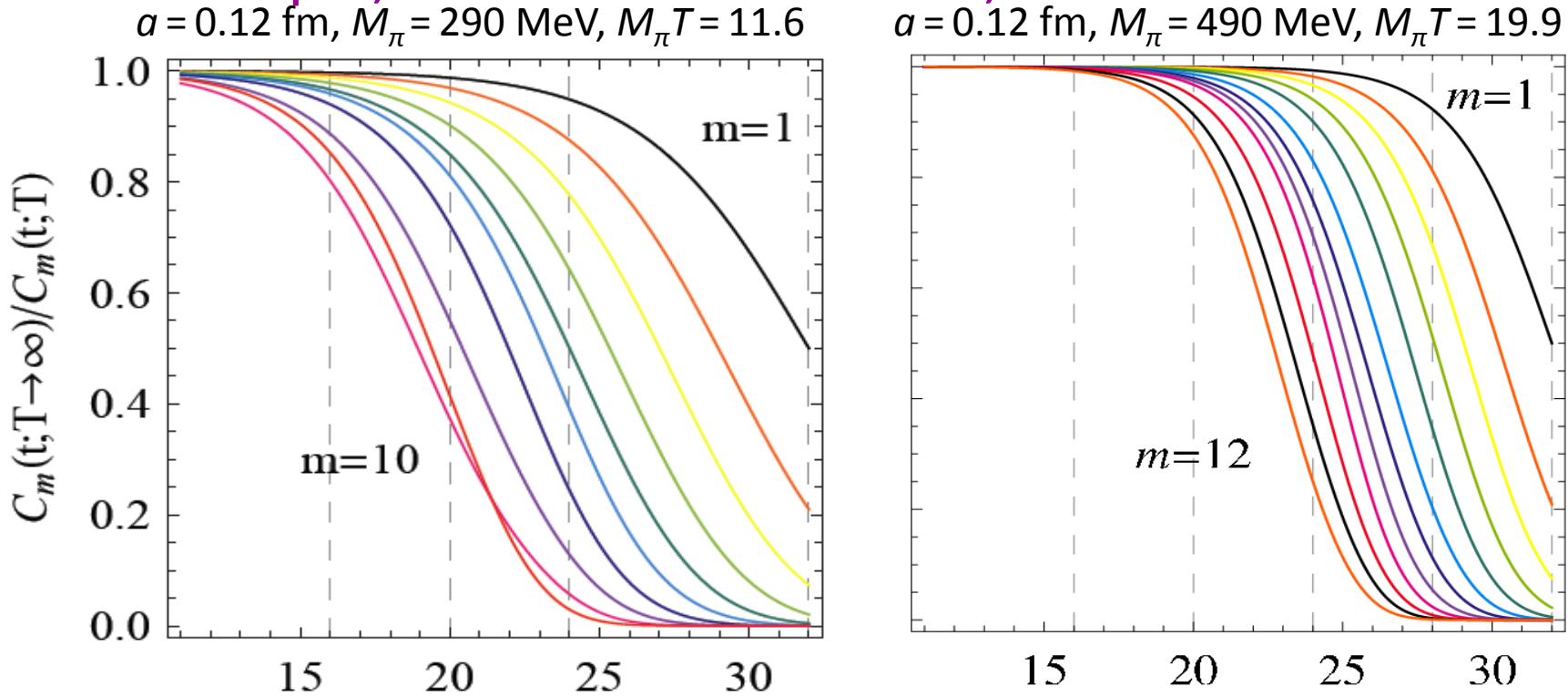
$$t_{\text{sep}} = T/2$$



Thermal Contamination

§ Significant for $n-\pi^+$ system through amplitudes

§ For example, $a=0.12$ fm ensemble, $T=64$



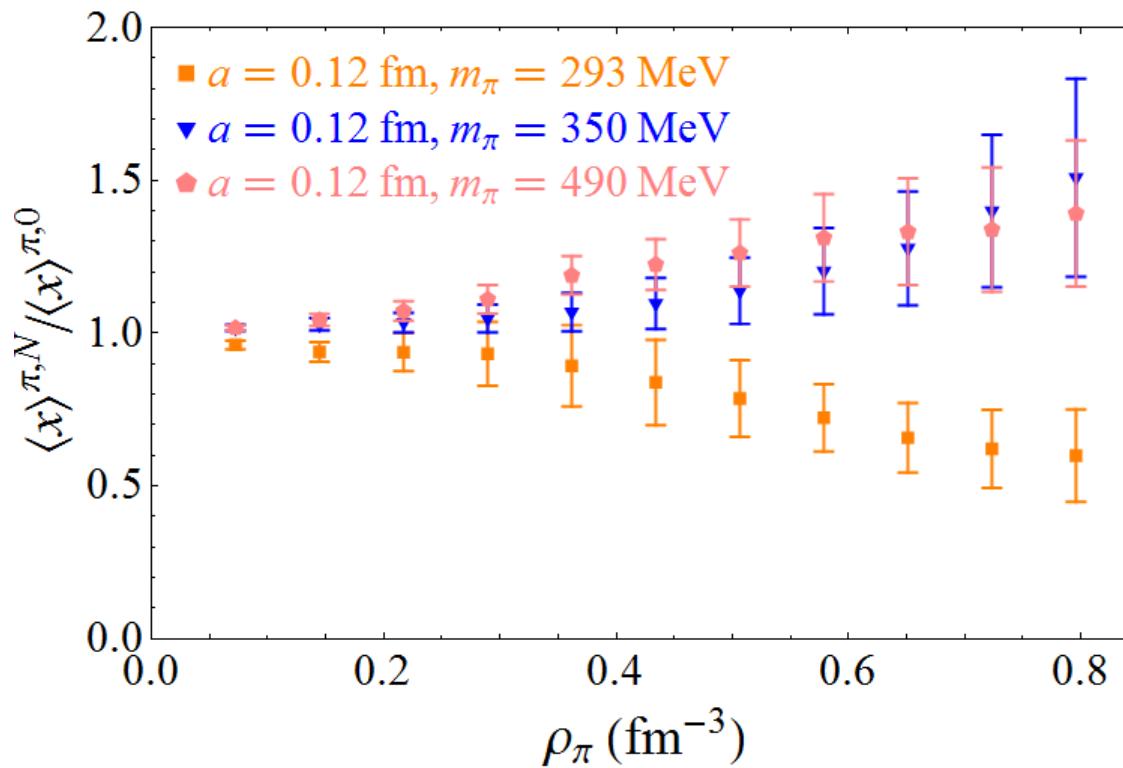
§ Matrix elements extraction for $p=0$ (in reality)

$$C_m^{(n)}(\tau, t, 0) \rightarrow \sum_{\ell=0}^m \binom{m}{\ell} Z_m^{(\ell)} \langle O_{m-\ell}^{(n)} \rangle e^{-E_{m-\ell} t} e^{-E_\ell(T-t)} + \dots$$

Pion Momentum Fraction

§ $\langle x \rangle^{\pi, N} / \langle x \rangle^{\pi, 0}$ with thermal-state degrees of freedom
multiple t_{sep} used

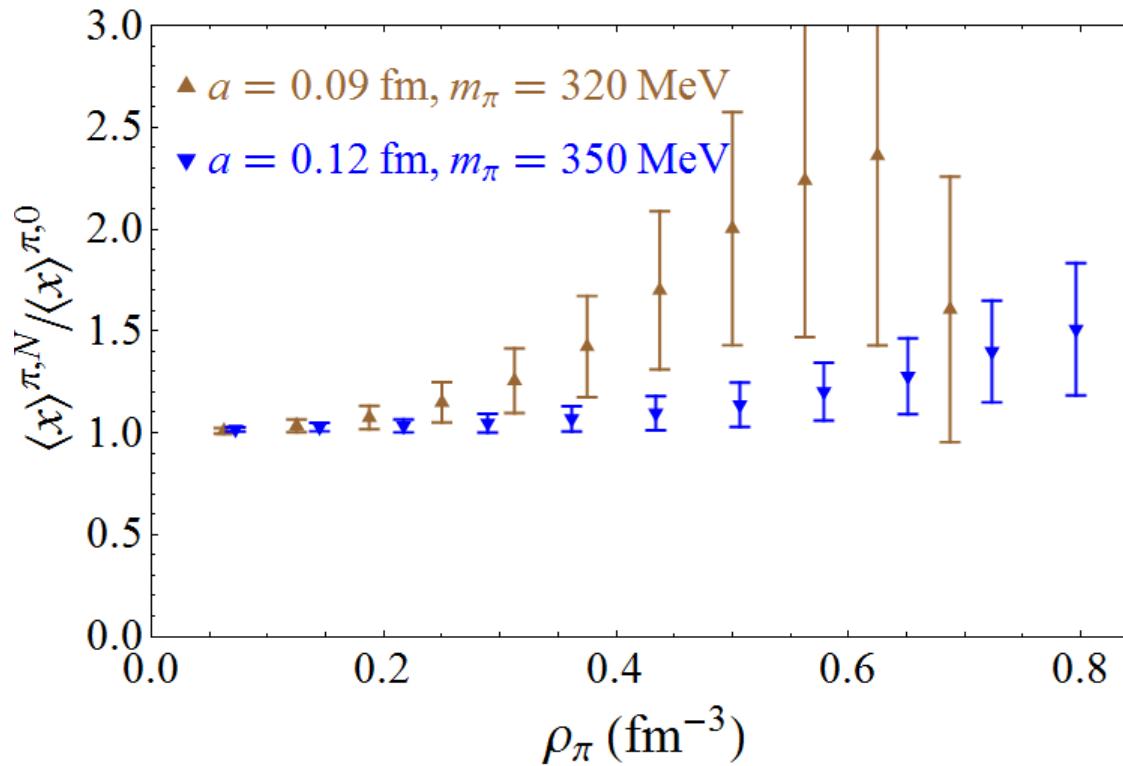
$$O = O_{\{44\}} - \frac{1}{3} (O_{\{11\}} + O_{\{22\}} + O_{\{33\}})$$



Pion Momentum Fraction

§ $\langle x \rangle^{\pi, N} / \langle x \rangle^{\pi, 0}$ with thermal-state degrees of freedom
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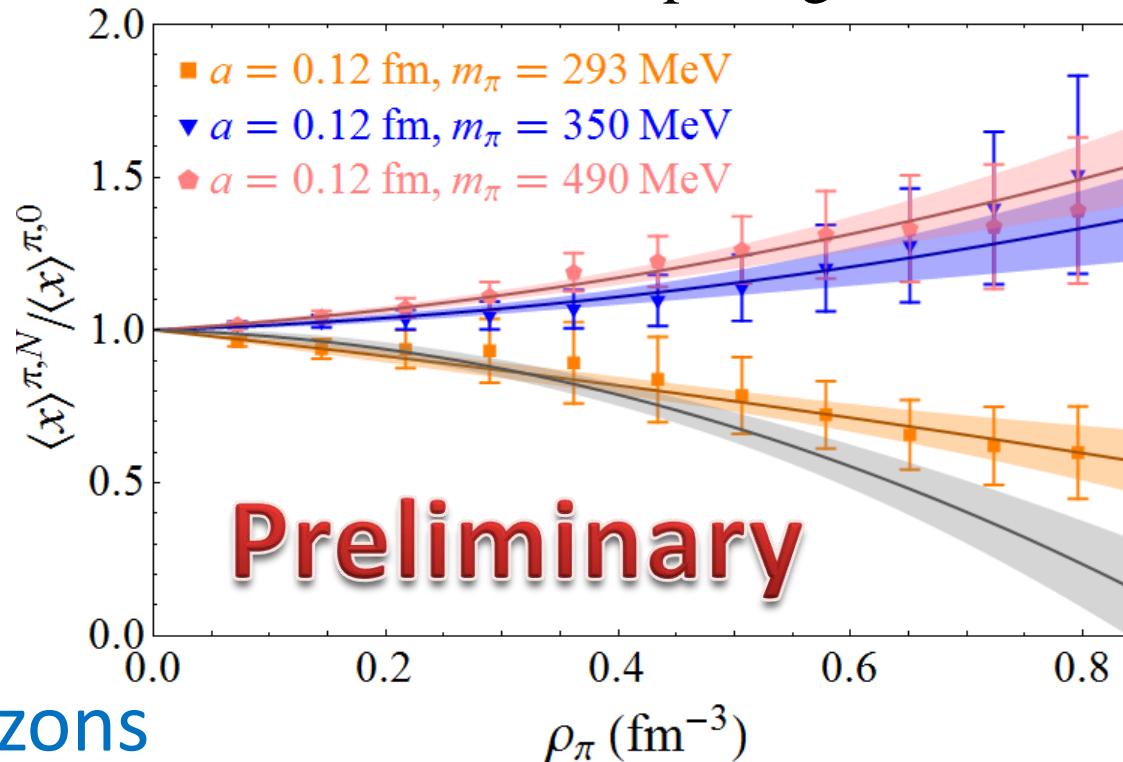
$$O = O_{\{44\}} - \frac{1}{3} (O_{\{11\}} + O_{\{22\}} + O_{\{33\}})$$



Summary and Outlook

§ First lattice-QCD attempt to measure EMC effects

- ❖ Pion momentum fraction in pion medium
- ❖ With $m_\pi \approx 290\text{--}490\text{ MeV}$, 2 lattice spacings



New Horizons

§ Nucleon structure in meson and light-nuclear media